c. Algebra and Number Theory,
d. Information Theory and Applied Mathematics,
e. Stochastics,
f. Foundation of Mathematics, Mathematical Logic and History of Mathematics.

2. To test the knowledge of selected subjects from applied mathematics specified for the course.

3. To test in depth the major subject of the candidate. A candidate is also expected to have some knowledge of the historical and current developments of mathematics. Those who offer mathematics as a minor subject are only required to sit for an oral examination. The oral examination is less demanding and its objective is stated in (1) above.

Generally both *Diplom* and *Staatsexamen* candidates look for their own examiners who have to be approved by the Department. The syllabus for each of the subjects to be examined is laid down by the examiners themselves.

Oral examination is a special feature in German universities. While it is always a difficult task for the examiner to find suitable problems for the oral examination, this examination system enables the examiner to test in depth a candidate's knowledge of a given subject. Also, the examiner may help the candidate by changing or modifying the question during the examination, which is not possible in a written examination.

Apart from having to attend lectures and related tutorials, the students have to participate in at least one seminar in the case of *Staatsexamen* candidates and in at least two seminars in the case of *Diplom* candidates. In a seminar, original research topics are distributed among the participants. The students may work individually or in groups and they take turns to present their papers. In most cases, problems for the thesis or academic exercise arise from the seminars. The professor in charge would distribute the thesis problems according to the performance of the students at the seminar. The thesis for *Diplom* is expected to be quite substantial,normally including some new results, and should show the candidate's ability to do independent research. The requirement for the academic exercise is lower. The courses leading to *Staatsexamen* are meant for those who want to become school teachers. These courses therefore include some subjects from didactic mathematics.

Under this apparently flexible system, a student has the distinct advantage of having more time to develop himself. On the other hand, he needs strict self-discipline and proper guidance, otherwise, he may face the danger of not having sufficient credits to do his *Vor-Diplom* or *Vor-Examen* A reform is underway to restrict the number of years that a student is allowed to spend in a university, and whether this is for better or for worse is still to be seen.

SECONDARY SCHOOL MATHEMATICS IN JAPAN

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The purpose of this study is to explain the current features of secondary school mathematics in Japan. But it is necessary, first, to create the context, namely the whole system of schooling. This will now be described. Where figures are quoted, the source is the Statistical Handbook of the Ministry of Education, 1978.

The system of schooling in Japan

Schooling is compulsory from the age of 6. It begins in a six-year elementary school. On completing the elementary course, all children move on to three years of lower secondary education. This sequence of nine years of schooling is compulsory for all, and the attendance rate is 99.9 per cent. Of those who complete the lower secondary course, the greater proportion go on to upper secondary schooling, where, in 1977, the attendance rate had reached 93.1 per cent.

In Japan, there are three types of higher institutions: university,junior college and technical college. Technical colleges require for admission the completion of lower secondary schooling, and offer a five-year training programme for technicians. At present, there are about 10,000 students in each grade. This is only 1 per cent of the graduates of the lower secondary schools. Graduates of the upper secondary schools may go on to the universities and the junior colleges.

Schools run by the central government, local government, or non-governmental co-operatives are respectively called national, public or private schools. The number of schools and of students and their relative proportions in 1977 are shown in Table 1.

As Table 1 shows, there are relatively few national or private elementary and lower secondary schools. Indeed, the number of pupils in these elementary schools accounts for 1 per cent or less of the total of the elementary school population, while in lower secondary schools this rises to a little over 3 per cent. But, at the upper secondary school level, private schools account for 24.4 per cent of the total, while the number of pupils they cater for is 28.8 per cent of the total. However, at the university and junior college level, the part played by private schools contrasts markedly with their earlier role. The consequences are, as is shown in a later section, severe competition in the examinations for entrance into institutions of higher education, notwithstanding the fact that the institutions are sufficient in number to absorb the students. Furthermore, these circumstances have an effect on the organization of courses of study in the secondary schools.

Table 1

		Total	National (percentage)	Public (percentage)	Private (percentage)
Elementary schools	Schools	24,777	0.3	99.1	0.7
	Students	10,819,656	0.4	99.0	0.5
Lower secondary schools	Schools	10,723	0.7	94.2	5.1
	Students	4,977,108	0.7	96.1	3.1
Upper secondary schools	Schools	5,028	0.3	75.3	24.4
	Students	4,381,062	0.2	71.0	28.8
Universities	Schools	431	20.4	7.7	71.9
	Students	1,839,363	20.8	2.8	76.4
Junior colleges	Schools	515	6.2	9.3	84.5
	Students	374,244	3.7	5.0	91.3

The existing curricula for the lower and upper secondary schools

The main features of education in the lower and upper secondary schools may be summarized as follows:

The general framework

The subjects to be offered, and the standard number of teaching hours per year for each subject in elementary and lower secondary schools, are laid down in the Ministry of Education Ordinance. For upper secondary schools, the same ordinance lists only the subjects.

The basic framework, in accordance with which each school organizes its own curriculum including the specification of the objectives and the standard content of the teaching in each subject, is outlined in the national *Course of Study*, a guide compiled by the Ministry of Education for each of the three school levels. The methods to be used in teaching the content are mostly left to the judgement of the teachers except for a few principal considerations which are mentioned in the *Course of Study*. Hitherto, the *Course of Study* has been revised at approximately ten year intervals.

When the *Course of Study* is due to be revised, the draft plans for a revision are made co-operatively by specialists in the Ministry of Education and a committee composed of scholars, intelligent laymen, school teachers, school principals and educational administrators, appointed by the Minister of Education, and announced officially by him for public criticism. Final decisions for a new *Course of Study* are made by the Minister, who takes into account the various suggestions set out in the draft plans.

Textbooks

Most of the textbooks used in schools are compiled by commercial publishers in accordance with the requirements of the *Course of Study*. They are then published for school use, providing they have the requisite authorization of the Ministry of Education. Each school adopts only one textbook for each subject. The choice from among the several available textbooks is left, in the case of the public elementary and lower secondary schools, to the municipal boards of education and, in other cases, to each school.

An overall view of the present *Course of Study* of mathematics in lower secondary schools

The present *Course of Study* was made public in 1969, and made effective as from 1972. In 1981, it will be superceded by the next *Course of Study*, revised in 1977.

The standard number of school hours (a school hour is 50 minutes) allotted to mathematics is four per week in each grade. The school year comprises thirty-five weeks or more.

The subject name for mathematics education in the lower secondary school is *Sugaku* (Mathematics). The subject is not divided into separate branches such as algebra or geometry. The teaching content is classified, grade by grade, into five areas as follows:

A. number and algebraic expression;

B. functions;

C. geometric figures;

D. probability and statistics;

E. sets and logic.

By the end of the ninth grade, all pupils in lower secondary school are expected to have covered the following content of mathematics:

(a) In area A, algebra, up to the solution of quadratic equations, within the domain of rational numbers and their associated square roots;

(b) In area B, an introduction to the concept of a function up to the simple quadratic function;

(c) In area C, intuitive concepts of space figures, properties of plane figures up to the theorem of Pythagoras, the angle theorems of circles by deductive methods, and some topological properties through an intuitive approach;

(d) In area D, the descriptive statistics, including the concepts of mean, standard deviation, and correlation exhibited diagrammatically, together with concepts of probability;

(e) In area E, concepts of sets and logic, including the terms and the symbols associated with them, as are needed to facilitate the learning of the content of the foregoing areas A, B, C, and D.

The broad objectives of lower secondary school mathematics

The general aim is to foster in pupils the ability to think mathematically and logically, to help them to grasp various phenomena, and to enable them to consider and to handle them in a unified and developmental way. To this end, the particular aims should be to help pupils:

1. to deepen their understanding of the basic concepts, principles, and rules relating to quantities,to geometric shapes, etc., so fostering their ability to develop more advanced ways of treating and disposing of these things in a mathematical manner;

2. to acquire the basic knowledge of, and to master the basic skills relating to quantities and geometric shapes, so developing the ability to make accurate and efficient use of these skills and understandings;

3. to help pupils towards a deeper understanding of the significance of using mathematical terms and symbols, so fostering in them the ability to use the terms and symbols simply and precisely when considering the nature of and the mutual relations between quantities, geometric figures, etc.;

4. to develop the ability of so seeing through a problem that they can consider it logically, and deal with it, presenting the solution in a manner which is consistent with the way in which the problem was presented.

The content and the objectives stated above are the same for all pupils, irrespective of the individual differences in their aptitude or their future career intentions. All pupils, therefore are expected to follow the same course throughout their three-year course of mathematics. In the *Course of Study*, it is suggested that individual differences should be catered for by differentiating the depth of treatment of the content. The range of topics listed above is, however, the same for all pupils.

An overall view of the present *Course of Study* of mathematics in upper secondary schools

The present *Course of Study* was made public in 1970 and became effective as from 1973. It is, however, scheduled to be replaced in 1982 by a new *Course of Study*, revised in 1978.

The subject name for mathematics education in the upper secondary school is also *Sugaki* (Mathematics). It is not, however, organized on a grade system, as it is in the elementary and lower secondary schools. Instead, the subject is organized according to a sequence of short courses, some being compulsory, others being optional.

The names of the various short courses that are on offer, and the standard number of school hours allotted per week to them, are as follows:

General Mathematics	6
Mathematics I	6
Mathematics IIA	4
Mathematics IIB	5
Mathematics III	5
Applied Mathematics	6

In addition, there is a specialized course, the so-called "Science-Mathematics" course, provided for pupils whose curriculum is specially biased towards natural sciences and mathematics. The course is made up of special short courses other than those listed. One of the ingredients, for example, is "Integrated Mathematics". This is studied for thirteen to eighteen school hours per week in the eleventh and twelfth grades, after taking Mathematics I in the tenth grade, and a course in "Computer Mathematics", which takes up two school hours per week for a year.

The rubrics which apply to the short courses listed in the penultimate paragraph are as follows:

1. All pupils are required to take either General Mathematics or Mathematics I; the others are optional;
2. Mathematics IIA, IIB, and Applied Mathematics should be studied after Mathematics I; and Mathematics III should follow Mathematics IIB.
3. The course comprising Mathematics I, IIB and III may be regarded as an academic one;
4. General Mathematics and Mathematics IIA may be regarded as a non-academic course, as the emphasis in these courses is on intuitive approaches and practical activities such as experiments or actual measurements;
5. Applied Mathematics is intended mainly for pupils in vocational courses, and for pupils who take a practical course; they learn advanced topics needed to support their specialized subjects.

In the academic course, comprising the sequence Mathematics I, Mathematics IIB and Mathematics III, topics from algebra, geometry, analysis, probability and statistics are integrated into a coherent system, with material in set theory and logic being appropriately related to them whenever possible. The detailed aims follow.

Algebra in Mathematics I

1. To understand the real numbers and the operations associated with them.
2. To realize that polynomials and rational expressions are similar in structure to integers and rational numbers respectively, and to learn to compute with simple polynomials and rational expressions.
3. To develop an understanding and an ability to work with quadratic equations and inequalities with real coefficients, and with simple absolute inequalities.
4. To deepen the understanding of numbers by extending the range to include real and complex numbers, and also to appreciate that equations can have their solutions in the extended range of numbers.

Algebra in Mathematics IIB

5. To understand matrices and their operations.
6. To understand that by using matrices a system of simultaneous linear equations can be expressed as one equation, and to understand how linear transformations are related to matrices.
7. To understand the principle of mathematical induction by applying it to the binomial theorem and to sequences of numbers; also, to understand the procedure and the significance of defining things inductively by stating the characteristics of a simple sequence of numbers.

Geometry in Mathematics I

1. To understand the concept of a vector in a plane, and to master the operations of addition and subtraction of vectors, and of multiplying a vector by a real number.
2. To learn how to describe a plane figure in relation to a co-ordinate system by means of an equation.
3. To learn how to study the properties of plane figures, and the relationships between them by referring to their equations.

Geometry in Mathematics IIB

4. To understand the role of axioms in mathematics and to understand the meaning of axio-

matic construction by a study of plane geometry.

5. To extend the notions of co-ordinate and vector systems to three dimensions, and to understand that vectors are based on the same concept both in the plane and in space.

Analysis in Mathematics I

1. To understand the meaning of mapping, and the composition of mappings and inverse mapping.
2. To perceive functions as mappings.
3. To understand the characteristics of simple functions, such as the quadratic function, simple rational functions, and exponential and logarithmic functions.
4. To understand the characteristics of trigonometric functions and their periodicity.

Analysis in Mathematics IIB

5. To understand the concepts of the differential coefficient and the derivative, and to be able to compute and to apply derivations of simple integral functions.
6. To understand the concept of integration, and to integrate simple integral functions and apply them to problems.

Analysis in Mathematics III

7. To understand the idea of a limit, as it applies to an infinite sequence of numbers, and thereby to understand the meaning of an infinite geometric series.
8. To be able to calculate the derivatives of simple elementary functions and apply them to problems, as follows:
   (a) Differentiation of the quotient of two functions, such as:
   $y = (ax + b) / (x + c)$ and $y = (ax + b) / (x^2 + 1)$;
   (b) Differentiation of composite and inverse functions, such as:
   $y = x^n$ (n is a rational number),
   $y = \sqrt{ax + b}$ and $y = k\sqrt{a^2 - x^2}$;
   (c) Derivatives of trigonometrical functions;
   (d) Derivatives of exponential and logarithmic functions, such as:
   $y = e^{ax}$ and $y = \log(ax + b)$;
   (e) The second derivative.
9. To deepen the understanding of integration, and to calculate the integrals of simple elementary functions and apply them to problems, as follows:
   (a) Integration by parts, but limited to only one such application;
   (b) Integration by substitution, but limited to such types as:
   $t = ax + b$ or $x = a \sin \theta$.
10. To appreciate the significance of differential equations, and to solve such equations as $dy/dx = ky$ (k is a constant).

Probability and statistics in Mathematics I

1. To understand the meaning of probability and its basic laws.
2. To become proficient in determining the probability of the occurrence of events in simple

situations.

Probability and statistics in Mathematics III

1. To gain a clear idea of the meaning of a population and a sample, and of the meaning of a probability distribution.
2. To be familiar with the binomial and the normal distributions, and to use them in simple cases.
3. To understand the basic ideas of statistical inference, such as estimation and test.

Sets and logic in Mathematics I

1. To deepen the understanding of sets, propositions, compositions and relations among propositions.
2. To relate the materials in sets and logic to the materials in other parts of the courses as background for facilitating the learning of their topics.

The broad objectives of upper secondary school mathematics

The main aim is to help pupils to develop their ability to formulate in mathematical terms problems which occur in a variety of phenomena, and to solve them in logical, systematic and integrated way. The course also aims at helping pupils to develop an appreciation of the importance of the role of mathematics in society. To this end, the specific aims should be to help students:

1. to gain an understanding of the fundamental concepts, laws, principles, and relationships in mathematics, so as to develop the ability to think about and deal with things in more advanced mathematical ways;

2. to acquire basic knowledge and to master basic skills combined with an ability to apply them efficiently and accurately;

3. to deepen their appreciation of the significance of using the terms and symbols of mathematics, and to develop their ability to express mathematical properties and relations concisely and clearly, thereby to promote a readiness to use them in real situtations;

4. to develop insight and the ability to think logically through abstract ideas, and, in the case of practical problems, to cultivate a readiness to check results for their reasonableness;

5. to understand the way in which a mathematical approach may be used in the systematic organization of procedure.

By scrupulously examining the above objectives of upper secondary school mathematics, they would be found to be a natural and coherent development of the objectives of lower secondary school mathematics already described.

The social background to the recent urge to reform the existing national curricula in Japan

As has already been pointed out, revisions have recently been made to the existing national curricula. These will come into effect as follows:-

The Elementary School (Grade 1 - 6) revised in 1977, to operate in 1980
The Lower Secondary School (Grade 7 - 9) revised in 1977, to operate in 1981
The Upper Secondary School (Grade 10 - 12) revised in 1978, to operate in 1982

Behind these changes are a number of social problems which deserve description.

The higher rate of school attendance

A rising rate of school attendance in compulsory education has been a national tradition since the beginning of the twentieth century. It has continued to the present, even though compulsory education was extended to lower secondary school by the post-war innovations of 1947. Almost 100 per cent of the age groups of compulsory education are now attending school.

Likewise, there has been a rising percentage in the numbers of those who go on to upper secondary schools after completing lower secondary schools, beginning with 42.5 per cent in 1950, increasing to 57.7 per cent in 1960, 82.1 per cent in 1970, and 93.1 per cent in 1977.

This trend has been followed by a rising rate of entry into universities or junior colleges, as follows: from 10.1 per cent in 1955 to 10.3 per cent in 1960, 24.0 per cent in 1970, 35.3 per cent in 1974, and 38.3 per cent in 1977.

These trends appear to reflect some correlation between popular belief in the importance of school education and popular awareness of a growth in economic development.

Inevitably, however, such sharply increasing attendance rates have introduced many low achievers into the higher levels of education, and the questions of how to educate them suitably has become a severe social problem.

Antipathy to differentiation and bias to academic courses

There has also grown up a popular antipathy to differentiated curricula, and, with it, a demand for equal opportunity to enter higher education courses, notwithstanding the wide difference in aptitudes among students. For example, in the curriculum revised in 1962, the mathematics laid down for the ninth grade was divided into 'required content' and 'selective content'. The intention was to make it possible for some pupils to limit their learning strictly to what was 'required'. But, in practice, very few schools were prepared to provide a course which covered the 'required' content only. Consequently, this provision was discarded when the time came for the next revision in 1969.

One more example may be cited. In upper secondary schools, the proportion of pupils who select each of the short courses in mathematics may be estimated by the demand for copies of textbooks. By this criterion, the options work out as shown in Table 2.

Table 2

Grade	Option	Percentage of students selection option
In the tenth Grade	General Mathematics	2
	Mathematics I	98
	Mathematics IIA	25
In the eleventh Grade	Mathematics IIB	50
	Applied Mathematics	10
In the twelfth Grade	Mathematics III	30

The above estimation indicates that the percentage of pupils who select non-academic short courses, such as General Mathematics and Mathematics IIA, is much lower than that of pupils who select academic short courses.

All of the above point to the prevalent tendency of parents and pupils wishing to follow an academic course in mathematics. Many people seem to think that it is good for their children to receive a university education if possible. Accordingly, courses which lead elsewhere are judged undesirable.

But,such extreme parental bias towards academic courses has laid upon pupils burdens which for some are too heavy to bear, especially the slow learners. How to deal with this state of affairs has become another severe social problem to be settled.

Heavy competition in the entrance examinations

Selection is by examination for entrance into national or private lower secondary schools from public elementary schools, into upper secondary schools from lower secondary schools, and into universities or junior colleges from upper secondary schools. Only for entrance into public lower secondary school from elementary school is there no selection procedure.

These entrance examinations are of a competitive nature in that a limited number of places are available and the competition for entrance into schools or universities with a high social prestige is accordingly very severe. Though the percentage of successful candidates quoted earlier might look higher than might be expected from what has just been said, it must be realized that the high figures result from candidates who failed to achieve their first choice being obliged to take offers from the second or third best.

As mathematics is usually one of the key subjects in these examinations, schools tend to prepare pupils for selection examinations in ways which militate against their providing a normal and desirable form of mathematics teaching.

With the public upper secondary schools, the examinations for entry are administered by each prefectural board of education, and are open to candidates from all the schools under its control. In other cases, each school, university, or junior college administers its own examinations separately, and sets problems independently. Commonly, when a new type of problem is set in some examination it is likely to become a fashion in other examinations in subsequent years. This tendency, in turn, has its effect upon lower schools. It augments the work done by way of preparation for examinations at the expense of the normal course of study. It also places such an importance upon the coverage of different types of problem solving as to result in a neglect of other important aspects of mathematics education.

Those who fail the examination for entry into their desired university usually continue, after graduation, to prepare to try again the following year. These students are called *ronin*, which means a masterless *samurai*, or feudal warrior in Japan. In many large cities, there are private schools, outside the regular system, who recruit the *ronin* and prepare them for the next examination. These schools are called *Yobiko* (preparatory school). Many of them are prospering.

In addition, tutorial classes, called *Gakushu Juku*, also private enterprises, are becoming popular in cities. In these classes, pupils of elementary or lower secondary schools can receive special tutoring, by attending outside their regular school hours, to avoid their becoming future *ronin*.

In order to mitigate the hardship presently prevailing in the competition for entry to the universities, a new national institution, called the University Entrance Examination; Center, was established in May, 1977, and a new system of entrance examinations, based upon its services, applied to all national and public universities throughout the country as from January 1979. Under this system, the Center will become responsible for the evaluation of the scholastic attainment of all candidates for admission to university in the general and basic learning content of upper secondary school education. The final decision to admit students will remain with each university. Some will decide after considering only the achievements of candidates as assessed by the Center; others will decide after adding to the Center's assessment the results obtained in a second examination administered by the university itself.

Efforts made to assist teachers to train themselves for the new existing course of study of mathematics, and social criticisms of the new curriculum

In revising the *Course of Study* of secondary school mathematics, a number of modern mathematical concepts were introduced. These included sets, mapping, probability, logic, structure, matrices and so on. With these innovations, it was expected that, when the aims were fully realized, further progress would have been made in mathematics education in Japan. In order to ensure the success of the curriculum reform, the following opportunities for teachers to train themselves were given:

(a) workshops were held on a nation-wide scale uner the auspices of the Ministry of Education;

(b) regional workshops were conducted by the Board of Education or the Educational Research Center in each prefecture;

(c) meetings were called to receive the results of research in schools designated by such authorities as the Ministry of Education or the Board of Education;

(d) educational television programmes were broadcast by Nippon Hoso Kyokai (NHK - Japan Broadcasting Corporation) to help teachers to understand the new curriculum through the demonstration of model lessons on the modern concepts and ideas followed by discussion among the participants in the programme;

(e) guide materials were prepared and edited by the Ministry of Education;

(f) national and regional meetings on mathematics education were arranged by such private study groups as the Japan Society of Mathematical Education (JSME).

As a consequence of these earnest efforts of teachers to improve their methods of teaching and to introduce the modern concepts, good results were expected. Unfortunately, severe criticisms of the new curriculum began to appear soon after it was made public. The background to this reaction may already be inferred from what has been said in a previous section. The various causes of discontent were:

(a) the new curriculum does not cater for the many low achievers in higher school education arising from the steep increase of school attendance rates;

(b) the constitution of the short courses in upper secondary school mathematics does not take into account the pupils' antipathy to differentiation or their bias towards following an academic course;

(c) the content of the new curriculum appeared to make such demands upon the pupils that more than ordinary efforts were expected from them.

Severe criticisms were especially levelled at the course of study of mathematics, both by parents and teachers, and by some mathematicians. The critics insisted that it was not adequate to introduce new mathematics into the curriculum - the ideas were neither basic, nor easy for pupils to learn.

Furthermore, such media of mass communications as the press, television and various weekly magazines frequently announced that the new curriculum was responsible for a diminution in the ability of pupils to perform calculations.

In order to meet this critism, the Ministry of Education set in motion a revision of the existing curricula of elementary and secondary schools.

General principles governing the revision of the secondary school mathematics curriculum, 1977 and 1978

The guidelines for revision adopted by the Curriculum Committe nominated by the Ministry of Education are as follows:

(a) to lighten the pupils' load of learning by reducing the number of school hours assigned to all subjects in lower secondary school by about 10 per cent on the average;

(b) to fit the content of the teaching into the fewer school hours according to the following principles:

1. by avoiding fruitless duplication of the contents of mathematics from elementary school to upper secondary school, and by teaching concentrically;

2. by transferring some content to a higher grade or by cancelling some;

(c) by laying stress upon basic knowledge and the basic skills of mathematics, so as to develop an ability to think mathematically and to tackle problems in a mathematical way.

The main improvements to the *Course of Study* for lower secondary school mathematics

The essential improvements may be summarized as follows:

(a) the statement of objectives for the subject is reduced to the following concise form in contrast to that in the existing *Course of Study*: "To deepen the understanding of the basic concepts, the principles and the laws ot quantities and of geometric figures, and to develop the ability to express and deal with situations in a mathematical way, and thereby to cultivate the ability to handle them efficiently".

(b) the former basis of sets and of logic is no longer mentioned in the *Course of Study*; it is left to teachers or to writers of textbooks to work these in themselves;

(c) some topics have been deferred, and will be treated in a higher grade as follows:

1.	inequalities:	from seventh grade to eighth grade
2.	general definition of a function:	from seventh grade to ninth grade
3.	descriptive statistics:	from seventh grade to eighth grade
4.	probability:	from eighth grade to ninth grade

(d) some topics have been dropped entirely from the syllabus of the lower secondary school:

1. structures of a set of numbers;
2. simultaneous linear inequalities in two variables;
3. the cubic function, e.g. $y = ax^3$;
4. inverse functions;
5. the transformation of geometric figures;
6. permutations and combinations;
7. expectation, standard deviation and correlation.

It should, however, be noted that almost all of the above has been transferred to some part of the *Course of Study* for upper secondary schools. In the newly revised *Course of Study* of lower secondary school mathematics, the prescribed number of school hours per week are 3, 4, 4 for the seventh, eighth and ninth grades respectively. There will thus be one hour less per week of mathematics in seventh grade than in the existing *Course of Study*.

The main improvements to the *Course of Study* for upper secondary school mathematics

The revised curricula, made public in June 1978, were originally in draft, so as to invite public critism. It is, however, expected, from past experience, that the Minister of Education will pronounce the draft plan with slight modification to be the new *Course of Study,* to become effective from 1982.

The essential improvements to the *Course of Study* may be summarized as follows:

(a) The statement of objectives for the subject is reduced to the following concise form, as in the *Course of Study* of lower secondary school mathematics: "To deepen the understanding of the basic concepts, the principles and the laws, and to develop the ability to consider and deal with the situations in a mathematical way, by systematic organization, thereby to cultivate the ability to handle them efficiently";

(b) The names of the short courses which compose the main course, and the number of allotted school hours per week are as shown in Table 3.

Table 3. Allocation of hours among Short Courses

Mathematics I	4
Mathematics II	3
Algebra and Geometry	3
Basic Analysis	3
Differential and Integral Calculus	3
Probability and Statistics	3

The rubrics which apply to the above course are specified as follows:

1. Mathematics I is compulsory for all pupils in the tenth grade; the others are optional;
2. any one of Mathematics II, Algebra and Geometry, Basic Analysis, and Probability and Statistics can, as a rule, be studied after Mathematics I;
3. Differential and Integral Calculus should, as a rule, be studied after Basic Analysis;
4. Mathematics II is intended to be a non-academic course, but pupils who complete it may be allowed to take other optional courses if they wish.

For the specialized 'Science-Mathematics' course, two short courses, named 'Science Mathematics' and 'Integrated Mathematics' have been prepared. The former course covers roughly the same ground as Mathematics I, but it includes some computer mathematics. The latter course comprises an integration of the contents of the optional courses in the ordinary course. In this course, however, pupils are trained, through being given assignments and by working in seminars, to become more self-reliant in their learning habits.

The aims and content of the revised short courses can now be described.

Mathematics I

(a) Aim: To depen the understanding of numbers, of algebraic expression and of geometric figures, and to master the basic knowledge and skills associated with them, developing thereby the ability to use them exactly in handling mathematical situations.

(b) Content

1. Numbers and Algerbraic Expressions:
   a) numbers and sets;
   b) integers, rational numbers, real numbers;
   c) polynomials, rational expressions;

d) quadratic equations, simple equations of higher degree, simultaneous equations;
e) quadratic inequalities;
f) algebraic expression and proof;
g) terms and symbols: discriminant, the imaginary number, *i*, complex number;

2. Functions:
a) quadratic functions;
b) simple rational and irrational functions;
c) terms and symbols: inverse funciton;

3. Geometric figures:
a) trigonometrical ratios: sine, cosine, and tangent;
b) the sine theorem; the cosine theorem;
c) plane figures and their equations: the coordinates of a point; the equations of straight lines; the equations of circles;
d) terms and symbols: sin, cos, tan.

Mathematics II

(a) Aim: to help pupils to understand the basic concepts, principles and laws in the broader field of mathematics associated with the content of Mathematics I, and thereby make them more aware of the role played by mathematics in society.

(b) Content:

1. Probability and Statistics:
a) permutations and combinations;
b) probability;
c) statistics;
d) terms and symbols: ${}_nP_r$, ${}_nC_r$, factorial n, n!, complementary event, expectation, standard deviation;

2. Vectors:
a) vectors and their operations;
b) applications of vectors;

3. Differential and Integral Calculus;
a) the meaning of the differential coefficient:
b) derivatives and their applications;
c) the meaning of integration;
d) terms and symbols: limit, 'lim', indefinite integral, definite integral;

4. Sequences:
a) arithmetic series;
b) geometric series;

5. Various Functions:
a) exponential function;
b) logarithmic function;
c) trigonometric functions;
d) terms and symbols: power, root, $\log_a x$, generalized angle;

6. Computer and Flow Charts:
a) the function of an electronic computer;
b) algorithms and flow charts.

Algebra and Geometry

(a) Aim: To help pupils to understand vectors and matrices, to cultivate their abilities to use them, and furthermore, to deepen their understanding of conics and of figures in space by developing their ability to use co-ordinates and vectors.

(b) Content:

1. Conics:
   a) the parabola;
   b) the ellipse and hyperbola;

2. Vectors in a Plane;
   a) vectors and their operations;
   b) the inner product of vectors;
   c) the applications of vectors: equations of straight lines, circles, etc.

3. Matrices:
   a) matrices and their operations;
   b) the inverse matrix;
   c) linear transformation and mapping;
   d) terms and symbols: A^{-1};

4. Space Figures:
   a) points, straight lines and planes in space;
   b) co-ordinate system in space;
   c) vectors in space including equations of straight lines, planes and spheres.

Basic Analysis

(a) Aim: To help pupils to understand sequences, the exponential function, the logarithmic function, and the trigonometrical functions, and to inculcate the basic ideas of differentiation and integration, there by developing their ability to use them in case of simple integral functions.

(b) Content:

1. Sequences:
   a) simple sequences: arithmetic series; geometric series; etc;
   b) mathematical induction;
   c) terms and symbols: 'Σ';

2. Functions:
   a) exponential function;
   b) logarithmic function,
   c) trigonometrical functions: generalized angled and circular measure; trigonometrical functions, their periodicity, and the addition theorems of trigonometrical functions;
   d) terms and symbols: power, root, $\log_a x$;

3. Variation of the values of a function:
   a) the meaning of differential coefficient;
   b) derivatives and their applications: derivaties of the sum and difference of functions, and of a function multiplied by a real number; tangent, increase and decrease of the values of function, velocity, etc.;
   c) integrations and their applications: indefinite integral, definite integral, area, etc.;
   d) terms and symbols: limit, 'lim'.

Differential and Integral Calculus

(a) Aim: To help pupils to understand the concept of a limit and to deepen their understanding of the concepts and the laws of differentiation and integration, thereby developing their ability to use them in the case of simple elementary functions.

(b) Content:

1. Limits:
   a) the limit of a sequence;
   b) the limit of values of function;
   c) terms and symbols: convergence, divergence, ∞;
2. Differentiation and its applications:
   a) derivatives: differentiation of the product and quotient of functions; differentiation of composite and inverse functions; derivatives of trigonometric functions; derivatives of exponential and logarithmic functions;
   b) applications of derivatives: tangent, increase and decrease of the values of function, velocity, acceleration, etc.;
   c) terms and symbols: natural logarithm, 'e' the second derivative, a point of inflection;
3. Integration and its applications:
   a) integration: the meaning of integration, integration by substitution and by parts in simple cases, the integration of various functions;
   b) applications of integration: area, volume, length of path, etc.; the meaning of a differential equation, including the solving of such equation as $dy/ds = ky$ where k is a constant.

Probability and Statistics:

(a) Aim: To help pupils to understand the concept of a limit and to deepen their understanding and to help them to understand the concept of a probability distribution, thereby developing their ability to think in statistical terms.

(b) Content:

1. The arrangement of data:
   a) the distribution of a variate;
   b) the representation of values and of measure of dispersion;
   c) terms and symbols: variance, standard deviation, 'Σ';
2. The number of possibilities:
   a) permutations and combinations;
   b) the binomial theorem;
   c) terms and symbols: ${}_nP_r$, ${}_nC_r$, factorial n, n!;
3. Probabiltiy:
   a) probability and its basic laws;
   b) an independent trial and its probability;
   c) conditional probability;
   d) terms and symbols: complementary event, exclusive event, independence, dependence;
4. Probability distributions:
   a) random variable and its probability distribution;
   b) binomial distribution, normal distribution;
   c) terms and symbols: expectation;
5. Statistical inference:
   a) populations and samples;
   b) the idea of statistical inference;
   c) terms and symbols: estimation, test.

As the above shows, the content of the revised *Course of Study* for upper secondary school mathematics is more concisely expressed than it is in the existing course of study. In addition, it is the intention of the authorities to encourage flexibility both of content and of time, so that either or both may be increased or decreased according to the abilities and the aptitudes of the pupils. As has been said, these courses have yet to secure the approval of the Ministry before they become mandatory in 1982. It remains to be seen, therefore, whether, and to what extent, the proposed *Course of Study* will find favour with the critics of the current courses, will be found workable by the teachers in the schools, and will be more acceptable to students, parents and employers. Time alone will tell.

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